Analysis of the DES and Its Implications for the Design of an Extended DES[1]

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by

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Abstract

The Data Encryption standard (DES) has achieved wide utilization, especially in the financial industry. Whilst DES is a standard, the design criteria used in its development have been classified by the US government. This paper reviews what is known about the design criteria for the S-boxes, P-boxes, and key scheduling in the current DES. It then indicates how this information could be used to design an extended scheme with a double length key. There are two main objectives in doing this. One is because of increasing doubts about the ability of DES to withstand an attack based on exhaustive key-space searches, using specialized hardware. The other is to develop an encryption scheme for which the design rules used are known, and hence open to analysis and criticism.

[1] this technical report is expanded version of, and supersedes an earlier report, TR CS87/8 which also appears in the Proceedings of IFIP Sec'88 [Bro89]. It reflects the material presented at that conference.

1. Introduction

With the advent of computers, data encryption schemes underwent a revolutionary change. The use of computers enabled the cracking of hitherto secure schemes, but also enabled the implementation of schemes which until then were too complex for use by a manual cipherclerk. Based on the work of Shannon [Sha49], a new family of cryptosystems, known as **Substitution-Permutation networks** were developed. These led to the development of a system known as Lucifer at the IBM Research Laboratories [Fei73], and subsequently to the Data Encryption Standard (DES) [NBS77]. It has since achieved wide utilization, particularly in the banking and electronic funds transfer areas, where it is now an Australian standard [ASA85].

However, there was a considerable amount of controversy at the time of its introduction, primarily with the choice of a 56-bit key [DiH77], [Hel79]. This was on the grounds that such a key length was small enough to be exhaustively searched on machines which could be constructed in the foreseeable future. With the march of computer technology, this risk appears to be rising. However, on all other accounts DES appears to be an excellent cryptosystem. No short-cuts have been found to aid in the cryptanalysis of DES other than by exhaustive key-space search. With the current significant use of DES (especially in banking), there is interest in designing and building a DES-type cryptosystem with an extended key length, for several reasons. These include:

- Doubts about the ability of DES to withstand an attack based on exhaustive key-space searches on specialized hardware (cf. the decision of the US NBS to withdraw certification of DES for all but banking applications). This may be overcome by increasing the key length.
- The difficulty in obtaining DES systems outside the US, due to export restrictions.
- The desire to develop a scheme for which all the design criteria used are known, and hence open to criticism.

This paper will discuss the design criteria used in the DES, both those reported in the literature, and those noted by the author, and will illustrate how they will be applied in building an extended scheme.

2. Description of the DES

2.1. Fundamental Concepts

There are two basic cryptographic primitives: **substitution** and **transposition** (or permutation). In a substitution function, each character is replaced by some other character (see Fig 1). The particular substitution used is selected by some **key**. If characters are represented by a small number of bits, then such a scheme may be defeated by enumerating all possible cases to find the substitution used. If made sufficiently large and a suitably complex function is used, such a scheme is in practice unbreakable, since it would require the enumeration of all $n!$ cases (where n is the number of possible characters). Unfortunately it is also unworkable, since the number of permutations, and hence keys, would be equally large.

In a transposition function, traditionally, a set of characters were permuted (rearranged) into a new order, again with the particular rearrangement selected by some key. In modern implementations, this becomes a permutation of bits or a wire-crossing (see Fig 2). Since the complexity of such an operation is only linear in the number of bits used, it may be easily broken by testing each bit in turn to find its new location. It also implies that it is easy to build such a

Fig 1 - Sample S-box

Fig 2 - Sample P-box

function for large numbers of bits. Because of this, the concept of **product ciphers** arose, whereby a series of substitution and transposition functions are combined to form a function which is much more secure than any of its components. Prior to the introduction of rotor machines, only simple product ciphers were possible. The development of rotor machines, and subsequently computers, changed this significantly.

Shannon [Sha49], in a key paper, described a particular class of product ciphers which used a number of small (and hence implementable) substitution boxes, termed S-boxes, connected by large permutation boxes, termed P-boxes (see Fig 3). He termed these **mixing transformations**, whereby the S-boxes provided **confusion** of the input, and the P-boxes provide **diffusion** of the S-box outputs across the inputs to the next stage. Such schemes are now termed **Substitution-Permutation (S-P) networks**. Two key properties of such S-P networks are the **avalanche** property, identified by Feistel [Fei73]; and the **completeness** property, identified by Kam and Davida [KaD79]. The avalanche property states that changing one input bit should result in approximately half of the output bits changing. The completeness property states that each output bit must be a complex function of every input bit. These properties appear to be important criteria for providing cryptographic strength in the resultant system. More formally, these may be defined (see

Fig 3 - Sample S-P Network, with Avalanche Characteristic

[WeT86]) as follows. A function f is said to have a good **avalanche** effect if:

> for each bit i, $0 \le i < m$, one can divide the 2^m plaintext vectors into 2^{m-1} pairs X and X_i such that each pair differs only in bit i; and when forming the 2^{m-1} exclusive-or sums, termed avalanche vectors
>
> $$V_i = f(X) \oplus f(X_i)$$
>
> find that about half of these sums are 1.

A function f is **complete** if:

> for each bit j, $0 \le j < m$ in the ciphertext output vector, there is at least one pair of plaintext vectors X and X_i, which differ only in bit i, for which $f(X)$ and $f(X_i)$ differ in bit j

For a practical scheme however, it is obviously necessary for the recipient to be able to decipher the message. This implies that one must be able to compute the inverse of the enciphering function. For the simple S-P network shown above, it would be necessary to provide a separate decryption stage incorporating the inverses of the substitutions and permutations used. This is expensive in terms of both hardware and software. It is preferable to be able to use the same hardware for both encryption and decryption. This may be done by an elegant scheme which partitions the input and output blocks into two halves, $L(i-1)$ and $R(i-1)$, and uses only $R(i-1)$ at each round i (see Fig 4). This half $R(i-1)$, along with a key $K(i)$, is used as input to the encryption function g (which incorporates one stage of the S-P network), and $R(i-1)$ is also copied to $L(i)$ for the next round. The output from g is exclusive or'd (XOR'd) with $L(i-1)$ to form $R(i)$ for the next round. Thus when decrypting, $L(i)$ is copied back into $R(i-1)$ where it is used as input to g in order to recover $L(i-1)$. Thus it can be used to recover the other half of the input upon decryption.

2.2. Description of DES

The basic DES enciphering chain is shown in Fig 5. In brief, the input block is permuted by an initial permutation IP, is then subjected to a complex key-dependent computation, and is finally permuted by IP^{-1}, the inverse of the initial permutation. On deciphering, the same process is used, except that the complex component is run backwards, using keys in reverse order, via the method shown above. In more detail, the input block X of 64 bits is first permuted by the initial permutation[2] IP, and then divided into two halves $L(0)$ and $R(0)$. The complex key-dependent function consists of sixteen rounds in which the left component $L(i-1)$ is exclusive-or'd (XOR'd) with the output of a non-linear function g whose input is both the right component $R(i-1)$ and part of the key $K(i)$. The halves are then interchanged to form the input for the next round (with the exception of the final round). Finally the output is permuted by IP^{-1} (the inverse of the input permutation IP) to yield the output ciphertext Y.

The cryptographic security of DES rests, in the main, on the complex function g. It is shown in more detail in Fig 6. Its purpose is to ensure that the final DES output vector Y (see Fig 5.) is a complex nonlinear function of the input, such that there is no easy way of deriving the input knowing only the output, short of enumerating all possible keys. In detail, in round i, $1 \le i \le 16$, the right component of the message input $R(i-1)$ is first expanded from 32 to 48 bits via an expansion function $E(R(i-1))$. These are then XOR'd with a 48-bit component extracted from the key $K(i)$ to form vector A. This vector is partitioned into eight 6-bit blocks which are used as input to eight substitution boxes (S-boxes).

Fig 4 - Invertible DES transform

[2] for details of the actual permutation used in IP (and all the other functions in the DES) see [ASA85], or any of the modern textbooks on cryptography (eg [SeP89], [Den82]).

Fig 5 - DES Enciphering

Fig 6 - DES Function g

Each S-box produces a 4-bit output, all of which are concatenated to form the vector B. This vector is permuted by P and then XOR'd with the left component $L(i-1)$ of the message. In more detail, each S-box may be regarded as being composed of four substitution functions (row functions or rows). The two (outer or selector) bits of the 6-bit input selects one of these four functions, which then operate on the four (inner or substitution) bits to give a 4-bit output. This output is then permuted by P. Together these form one stage of an S-P network. The repetition of this procedure sixteen times forms the full S-P network.

The other critical component in the DES algorithm is the **key scheduling** stage. It is responsible for forming the sixteen 48-bit sub-keys used in the rounds of the encryption procedure. It is shown in Fig 7. This function is important since if the same key is used on successive rounds, it can weaken the resulting algorithm. This trap is mostly avoided with the specified key scheduling algorithm, except for a small number of keys known as **weak** or **semi-weak** keys (see [GrT78], [MeM82], [MoS86], [MoS86], and [ASA85]). In detail, the 64-bit key is permuted by $PC1$. This permutation performs two functions, first it strips the eight parity bits out, and then distributes the remaining 56 bits over two 28-bit halves $C(0)$ and $D(0)$. Subsequently these form two distinct paths. At each round, each 28-bit register is rotated left either one or two places according to the following schedule:

Table 1 - Key Schedule for DES

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Shift	1	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1
Total	1	2	4	6	8	10	12	14	15	17	19	21	23	25	27	28

After the shift, the resultant 28-bit vectors are permuted by $PC2$ (which in fact consists of two 28-bit permutations, not one 56-bit permutation), and are concatenated together to form the sub-key for that round. Note that the total number of shifts is 28; hence $C(16) = C(0)$, and $D(16) = D(0)$. When decrypting, it is necessary to generate the keys in reverse order. Because the final vector is the same as the initial vector, this can be done by rotating right, reading the table in reverse order (and with no shift before generating the first key). Thus, with minor changes, the same hardware is used for both encryption and decryption.

Fig 7 - DES Key Scheduling

3. Known DES Design Criteria

All the functions used in the DES are specified in tabular form. No description is provided on why they are chosen. In fact it is stated that the design criteria are classified. Nonetheless, some of the criteria have been released, and others may be deduced by examining the tables as given. The search for the design criteria is motivated by two reasons, first to aid in the cryptanalysis of the DES, and second in designing new and extended DES style schemes. The S-boxes and P-boxes, being the major components of the function g, will be examined independently.

3.1. S-box Design Criteria

The S-boxes in DES are the prime source of complexity in the algorithm, and are largely responsible for the security of the scheme. Consequently, considerable time has been spent on analysing them, and hence more is known about them than the P-boxes. The actual criteria used are classified secret. It has been stated[3] that there were 12 (possibly 13) criteria used, which resulted in about 1000 suitable S-boxes, of which the implementors chose 8. Some of the criteria used were reported to an NBS workshop on cryptography [BGK77], and are:

- 1 Each row function in an S-box is a permutation of the integers 0 to 15
- 2 No S-box is a linear or affine function of the input.
- 3 Changing 1 input bit to an S-box results in changing at least 2 output bits
- 4 $S(x)$ and $S(x \oplus 001100_2)$ must differ in at least 2 bits (where $S(x)$ is the output of an S-box given input vector x).

The following rules were labelled (see [BGK77]) as resulting from design criteria:

- 5 $S(x) \neq S(x \oplus 11ef00_2)$ for any choice of e or f.
- 6 The S-boxes were chosen to minimize the difference between the number of 1's and 0's in any S-box output when any single input bit is held constant.

These criteria and rules were analysed by Brickell et al. [BMP87]. Criterion 1 states, in effect, that each row of each S-box is a 1-1 function. If this were not true, then information would be lost, and recovery of the input would be impossible. Criterion 2 ensures that the DES algorithm as a whole is not linear or affine. If it were, it would substantially reduce the security of the scheme, and enable reasonably easy recovery of a key with a set of plaintext/ciphertext pairs. The next two criteria 3 and 4, seem to incorporate the concept of the avalanche effect, in that it ensures that small changes to one or two bits of input result in large changes in the output. Rule 5, reduces the probability that two different inputs would be mapped onto the same output (nb: function g is neither 1-1 nor *onto*, it must only generate the same output from the same input and key); and that the image space (ie: the range of possible values generated by g) is as large as possible. Criteria 1, 3, and 4 seem to imply rule 6 (which accords with the original statement that it is a consequence of the design criteria), and is related to the S-boxes being permutation functions.

Another consequence of design criteria was noted by Meyer and Matyas in their book [MeM82], namely that:

- 7 the S-boxes chosen required significantly more logical minterms[4] to implement than a random choice would require.

A random choice of S-box would lead to a distribution with a mean around 32. However Meyer and Matyas stated that as they added more design rules, the average number of minterms rose from around 46 in the early stages, to around 56 in the final design (with the distribution mostly ranging from 52 to 59). The final choice of 8 boxes was taken from the low end of this range (52 and 53) in order to make hardware implementation easier. This rule would indicate that the S-boxes are considerably more complex than a random choice would provide. It also implies that cryptanalysis of DES via formal coding techniques is extremely difficult, as was illustrated by Schaumuller-Bichl [Sch82].

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[3] by Alan G. Konheim at a Seminar series on Cryptography in Amsterdam, 1986 [Kon86]

[4] a minterm is a logical AND (boolean product) of input bits (and their negations), which form a necessary (but not sufficient) condition for a particular output bit to be asserted. An output bit may have its value completely described by the logical OR (boolean sum) of all its minterms. The number of minterms in such an expression (after simplification) is a measure of its complexity. In the worst case, for n input bits, 2^n minterms may be needed to describe each output bit.

3.2. P-box Design Criteria

There are a number of permutation networks in the DES design, IP, IP^{-1}, E, P, $PC1$, and $PC2$. They appear to be performing various functions. No design criteria for them have been released. However by examination of the various networks, certain features can be deduced. In general these all appear to be motivated by the avalanche and completeness criteria, in that there is a wide dispersion of outputs across the inputs to the next stage. We will examine these networks in turn.

3.2.1. IP and IP^{-1}

These two permutations form the initial and final steps respectively in the enciphering chain for DES. It has been argued [DDF83] that these permutations have no cryptographic significance, since exhaustive plaintext searches can be conducted on messages after these transformations have been applied. This argument probably does not hold when DES is used in one of its chaining modes. Other reasons put forward for their inclusion are that hardware implementations would benefit, and that it might help thwart other forms of attack. Further evidence for this may be found in Schaumuller-Bichl [Sch82] who show's that the complexity of DES depends mainly on the S-boxes, key scheduling and number of rounds, rather than the permutations. It was also noted that IP is extremely regular. As is shown in [HGD84], if the input block is broken into 8-bit bytes, then it may be implemented as eight iterations of an 8-bit permutation {2,4,6,8,1,3,5,7} into eight shift registers: a much more compact hardware realization, than a true 64-bit permutation would imply. Its functionality seems to be to distribute bits from each input byte over all eight bytes in the input block. Also it sends all even numbered bits to the left hand vector, and odd numbered bits to the right hand vector.

3.2.2. $PC1$

This transformation performs a very similar function to IP, and additionally strips off the parity bits to reduce the 64-bit key value entered down to the 56-bit key actually used. It is nearly as regular as IP, except for an asymmetry around the two 28-bit halves. As with IP, it may be implemented with eight iterations of a 7-bit permutation {0,1,2,3,4,5,6,7} (the parity bit 8 is discarded). However now the permuted value is read out by shifting two 28-bit halves toward the centre, rather than to one end, as in IP (see [HGD84]).

3.2.3. P and E

It has been noted by Davies [Dav82] and Davio et al [DDF83], that P and E need to be analysed together as the functional composition $P.E$. In fact $S.P.E$ should be analysed thus, being one round in an S-P network, since

$$R(i) = L(i-1) \oplus P(S(E(R(i-1)) \oplus K(i))).$$

The expansion function E is very regular. If the 32-bit input block is broken into eight 4-bit blocks, then the 6-bit output block is formed by concatenating the adjacent bit from the neighbouring blocks. If the 6 input bits to an S-box are labelled $abcdef$, then

- bits ab and ef are replicated in the neighbouring blocks and include the S-box selector bits for both the current (bits a and f), previous (bit b), and next (bit e) S-boxes,
- bits cd are substituted in the current S-box only.

Thus E serves to provide some diffusion by spreading the input bits at each round into adjacent S-boxes.

Now consider the composition $P.E$. The input of P is divided into 4-bit outputs from the eight S-boxes, and the output from E is divided into 6-bit inputs to the eight S-boxes at the next stage. Provided the S-boxes fulfil their design requirements, it may be assumed that all S-box outputs are equivalent, and that the inputs may be considered in the three pairs indicated above. Instead of expressing this permutation in terms of bit positions, it may be written in terms of which S-box output is connected by $P.E$ to each input bit at the next stage (see Table 2).

Table 2 - P.E Permutation							
S-box	Inputs from						missing box
	a	b	c	d	e	f	
1	7	4	**2**	5	6	**8**	3
2	6	8	**3**	7	5	**1**	4
3	5	1	**4**	6	7	**2**	8
4	7	2	**5**	8	**3**	1	6
5	3	1	2	**6**	**4**	8	7
6	4	8	**7**	1	3	**5**	2
7	3	5	4	**8**	2	**6**	1
8	2	6	3	**1**	**7**	4	5

From this form, the author notes that:

- 1 each S-box input bit comes from the output of a different S-box
- 2 no input bit to a given S-box comes from the output of the same S-box
- 3 an output from S(i-1) goes to one of the *ef* input bits of S(i), and hence via E an output from S(i-2) goes to one of the *ab* input bits.
- 4 an output from S(i+1) goes to one of the *cd* input bits of S(i).
- 5 for each S-box output, two bits go to *ab* or *ef* input bits, the other two go to *cd* input bits. This was noted in Davies [Dav82].

These rules all appear consistent with the implementation of the avalanche and completeness effects. They ensure that every output bit becomes a function of each input bit in as few rounds as possible. Meyer [Mey78] (also in [MeM82]) has shown that with the networks specified in DES, then after 5 rounds, every output bit is a function of all input bits. This analysis is repeated an extended in the next section. This result would appear to be an important consequence of the design criteria.

Permutations satisfying these criteria were generated. First rules 1 and 2 imply that each S-box takes inputs from 6 S-boxes. Hence one S-box apart from itself must have been missed. There are 264 ways possible to do this, which I termed exclusion sets. Then rules 3, 4 and 5 were used to generate classes of possible permutations[5]. A total of 178 such classes were found from 96 of the 264 exclusion sets (the others generated no valid permutations, due to the missing S-box being one of those required by rules 3 or 4). Each of these classes could generate a number of possible permutations by swapping the order of bits in each of the pairs *ab*, *cd*, and *ef*; or by rearranging the order of the four output bits from each S-box.

It is worth noting that both Davies [Dav82] and Davio et al [DDF83] have commented that the actual order of output bits from each S-box is irrelevant. This is because a functionly equivalent $S.P.E$ combination can be formed by rearranging the order of bits in each S-box (by rearranging columns), and then altering permutation P to compensate. As Hoornaert et al [HGD84] show, this is desirable in order to design an efficient hardware implementation which minimizes the number of wire crossings (and hence area) in P. It certainly illustrates that any analysis of these components needs to be performed on them as a whole.

3.2.4. *PC*2

The permutation $PC2$ is responsible for selecting 48 of the 56 key bits available on each round. The output of this permutation can be analysed in terms of the eight 6-bit blocks which become S-box inputs after being XOR'd with the expanded message block. As with the other components in the key scheduling stage, it can be regarded as two distinct 28 to 24-bit selections and permutations, with no interaction between the two halves. Again, like the permutation P, $PC2$ needs to be analysed in conjunction with the other components of function f, and the key scheduling function KS. We have

$$R(i) = L(i-1) \oplus P(S(E(R(i-1)) \oplus PC2(KS(U, i))))$$

[5] the mandatory wirings required by rules 3 and 4 were arbitrarily assigned to bits *c* and *e*/*a* respectively.

$$where\ U = PC1(K)$$

The author has noted that in each 6-bit output block corresponding to an S-box input, if the bits are sorted into ascending order of their input bits, then:

- 1 bits permuted to the same S-box input are no closer than 3 bits apart
- 2 bits permuted to an S-box input must have a span from lowest to highest input bit number of at least 22 of the 28 bits in each key half (alternatively, the average spacing must be at least 3 2/3)
- 3 bits permuted to the selector bits a, f on a given S-box must not be adjacent in the sorted list of input bits
- 4 bits not selected by PC2 must be at least 3 places apart

Rule 1 implies that the same bit of the key will not be used as input to the same S-box on successive rounds (since in the key schedule, bits are shifted only 1 or 2 places). Rule 2 results from application of the avalanche and completeness effects to ensure that the output depends on all key bits in as few rounds as possible, again as confirmed by Meyer [Mey78] (also in [MeM82]). Rule 3 ensures that the same bit is not used as a selector bit on successive permutations to an S-box. Rule 4 ensures that the same bit will not be missed on successive rounds (ie: that all keys bits are used as soon as possible).

3.3. Key Scheduling

Aside from the S-boxes and P-boxes, the other component critical to the security of DES is the **key scheduling** algorithm (see Schaumuller-Bichl [Sch82]). Several authors [Ber82], [Dav82], [QDD86] have noted that DES in fact uses a 768-bit key (formed from a 48-bit key to each of 16 stages), which is generated from the 56-bit key supplied. If either the key scheduling is poor [GrT78], or a bad choice of key is made [Dav82], then the same subkey might be repeated for more than one round, leading to insecurities in the scheme. No design rules have been published for the scheme currently in use. The design of the key scheduling function is obviously related to the design of $PC2$ by rules 1 and 4 given above. The author notes that the key scheduling function ensures that:

- 1 each bit is used as input to each S-box
- 2 no bit is used as input to the same S-box on successive rounds
- 3 the total number of bits rotated is 56 (which implies that $K(0) = K(16)$, enabling the decryption operation to use right shifts in reverse order).

Suggestions for alternate forms of key scheduling have been made in [Ber82] and [QDD86].

4. Ciphertext Dependency Analysis

One technique to provide a measure of the ability of a given S-P design to fulfil the completeness effect, is to analysis how rapidly the output bits become dependent on every input and key bit. This analysis was first performed by Meyer [Mey78] (also in [MeM82]). A basic assumption in this analysis is that each S-boxes ensures that every output bit was a complex function of all its input bits.

4.1. Ciphertext Dependence of Plaintext Bits

To provide a measure of this dependency, a 64 * 64 array $G_{a,b}$ is formed. Each element $G_{a,b}(i, j)$ of which specifies a dependency of output bit $X(j)$ on input bit $X(i)$, between rounds a and b. The number of marked elements in $G_{0,r}$ indicates the degree to which complete dependence was achieved by round r. Details of the derivation of this matrix, and the means by which entries are propagated, may be found in [MeM82]. The author repeated this analysis for the current DES, and extended it by analysing each of the 178 candidates for permutation P, as well as for a worst case P (see Table 3).

Table 3 - Worst Case DES P			
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8

The results are summarized in Table 4, and lead to the following conclusions:

- the current DES P results in complete ciphertext-plaintext dependence after 5 rounds.
- all 178 candidate permutations P result in complete dependence after 5 rounds.
- the worst case permutation P results in complete dependence after 7 rounds. Here E alone is relied upon to provide diffusion.

Table 4 - Dependency of Ciphertext bits on Plaintext bits in DES

Round	Std P	Candidate P	Worst P
1	6.25	6.25	6.25
2	32.06	32.03-32.10	21.09
3	73.49	73.44-73.58	43.75
4	96.90	96.87-96.95	68.75
5	100.00	100.00	89.06
6	100.00	100.00	98.44
7	100.00	100.00	100.00
8	100.00	100.00	100.00

The P-box used in the current DES has a propagation profile that falls fairly close to to the median of the 178 classes I have identified. It thus appears to be a fairly representative example of them. In conjunction with the substantially inferior profile of the worst case P-box, this also provides a strong indication that the design rules identified are comprehensive, at least as far as this aspect of the P-box design is concerned.

4.2. Ciphertext Dependence on Key Bits

This analysis is substantially more complex, since it is dependent on the choices of permutations P and PC2 as well as the KS the key schedule. To date, this dependency has been analysed for each of the P permutations used above, with the current key schedule, and with the current, worst case, and a sample regular PC2. To quantify this dependency, a 64 * 56 array $F_{a,b}$ is formed, each element of which specifies a dependency of output bit $X(j)$ on key bit $U(i)$. The vector U is that formed after PC1 is applied, ie $U = PC1(K)$. The number of marked elements in $G_{0,r}$ will be examined to provide a profile of the degree of dependence achieved by round r. Details of the derivation of this matrix, and the means by which entries are propagated, as before, may be found in [MeM82]. The author repeated this analysis for the current DES, and extended it as indicated before. The results are summarized in Table 5 for the current PC2, Table 6 for a worst case PC2, and Table 7 for a sample PC2. The sample PC2 was designed to satisfy the current rules known, and to use a regular wiring. It is listed in Table 8.

These results have led to the following conclusions:

- there are two errors in Meyers analysis. Firstly in his key schedule implementation he appears to have shifted, rather than rotated key bits. This alters the "type" of dependence, but not the number of dependent bits. Secondly, he appears to have miscalculated the dependence percentage. This same error was made in his cipher-plaintext dependence analysis in his original paper, but was corrected in his book. This did not occur in this analysis, and I suspect the same systematic error was at fault.

- the current DES PC2 results in complete ciphertext-key dependence after 5 rounds for current candidate permutations P, and in 7 rounds for the worst case permutation P.
- the worst case DES PC2 results in complete ciphertext-key dependence after 7 rounds for current candidate permutations P, and in 9 rounds for the worst case permutation P. Here *E* along with the key schedule are relied upon to provide diffusion.
- the sample DES PC2 results in complete ciphertext-key dependence after 5 rounds for current candidate permutations P, and in 6 rounds for the worst case permutation P. Interestingly, the profile is slightly worse for the current and candidate permutations P, but significantly better for the worst case.

Table 5 - Dependency of Ciphertext bits on Key bits in DES with Current PC2

Round	Std P	Candidate P	Worst P
1	5.36	5.36	5.36
2	39.17	38.39-39.29	24.67
3	82.25	81.25-82.48	53.12
4	98.44	98.21-98.55	79.69
5	100.00	100.00	95.65
6	100.00	100.00	99.78
7	100.00	100.00	100.00
8	100.00	100.00	100.00

Table 6 - Dependency of Ciphertext bits on Key bits in DES with Worst Case PC2

Round	Std P	Candidate P	Worst P
1	5.36	5.36	5.36
2	42.19	42.19	21.65
3	81.47	81.47	43.97
4	91.29	91.29	67.41
5	96.21	96.21	86.61
6	99.55	99.55	95.31
7	100.00	100.00	97.99
8	100.00	100.00	99.55

Table 7 - Dependency of Ciphertext bits on Key bits in DES with Sample PC2

Round	Std P	Candidate P	Worst P
1	5.36	5.36	5.36
2	38.62	38.39-38.84	22.77
3	81.47	81.25-81.70	48.66
4	98.21	98.21	75.45
5	100.00	100.00	94.20
6	100.00	100.00	100.00
7	100.00	100.00	100.00
8	100.00	100.00	100.00

Table 8 - Sample DES PC2					
1	5	10	15	20	25
2	6	11	16	21	26
3	7	12	17	22	27
4	8	13	18	23	28
29	33	38	43	48	53
30	34	39	44	49	54
31	35	40	45	50	55
32	36	41	46	51	56

5. An Extended DES

Having analysed the existing DES, and determined as many of the design rules as possible, a proposed design of an extended DES style scheme may be outlined. In doing this it is proposed to extend the work of Morris-Yates [Mor86], who used a subset of the criteria detailed previously. It involves widening all the data paths to twice the current width, and extending or redesigning all the S-boxes, P-boxes, and key scheduling. The initial motivation for doing this was to overcome the doubts about the ability of DES to withstand an attack based on exhaustive key-space searches by doubling the key length. This raises the effort needed from 2^{55} trials to 2^{111} trials[6], an impossibility in the foreseeable future. It will also result in a scheme for which all the design criteria used are known, and hence open to criticism; and which is not bound by foreign government restrictions.

5.1. S-box Redesign

Since sixteen S-boxes will be required in the extended scheme, an additional eight boxes will be required. One possibility is simply to reuse the existing eight boxes, either by using them twice, or by designing an additional eight only. These options were rejected both for copyright reasons, and because of doubts about the cryptological security of the current S-box 4 [BMP87], [Sha86]. Hence it was decided that sixteen new S-boxes will be generated. Ideally all possible 6*4 bit functions would be generated and evaluated against Both the known design rules, and additional criteria based on non-linearity criteria suggested by Pieprzyk [PiF89] and Carroll[7]. A subset will then be selected from those which satisfy these criteria. Because of the time this would take, it is envisioned that a random selection of boxes based on the non-linearity requirement will be generated and tested against the remaining criteria. This will be done both to generate suitable boxes for the proposed design, and to extend work (see [BMP87]) on the analysis of the design rules, identifying both redundant rules and possibly new rules.

5.2. P-box Redesign

The use of double width data paths implies a complete redesign of all the permutations. IP, IP^{-1}, and $PC1$ may be easily extended because of their regular design, if they are included at all. Morris-Yates [Mor86] has argued that because of doubts about their cryptological usefulness, they need not be provided at all (with the exception of the parity stripping component of $PC1$), saving a significant portion of silicon real-estate. If they are provided then the efficient implementation suggested by Hoornaert et al [HGD84] should be used. Initially, I will not be using these functions in the extended scheme.

The expansion function E may also be easily extended because of its regular design. This leaves the important permutations P and $PC2$. Because these are central components of the S-P network, and key scheduling respectively, they are critical to the security of the new scheme. As with the S-boxes, a sample will be randomly generated and tested against the known rules, in an effort to isolate any further design criteria, and to identify $S.P$ combinations which are functionally identical. It is here that this work differs most from Morris-Yates, who chose purely random permutations. Initially these will be specified as follows:

- P will use the existing design rules, but will have a regular wiring array. eg one proposal has inputs for $S(i)$ from $S(i-2)$, $S(i+1)$, $S(i+8)$, $S(i+5)$ (see Table 9). Such a regular array leads to efficient implementation in both

[6] there is a saving of half resulting from the well known symmetry in DES, namely that $DES(\bar{X}) = \overline{DES(X)}$

[7] in personal communications.

hardware and software.

Table 9 - Sample Extended DES P

15	2	9	6
16	3	10	7
1	4	11	8
2	5	12	9
3	6	13	10
4	7	14	11
5	8	15	12
6	9	16	13
7	10	1	14
8	11	2	15
9	12	3	16
10	13	4	1
11	14	5	2
12	15	6	3
13	16	7	4
14	1	8	5

- PC2 will use the existing rules modified to suit the larger shifts of 3 or 4 places needed in the Key Schedule. This implies that the constant 3 in rules R1 and R4 will become 5 in the initial design. Again as in the case of P, a regular wiring will be used, though it is complicated by the need to select a subset of the bits. One example distributes a bit to each S-box in turn, skipping every 7th bit (see Table 10).

Table 10 - Sample Extended DES PC2

1	10	20	29	38	48
2	12	21	30	40	49
3	13	22	31	41	50
5	14	23	33	42	51
6	15	24	34	43	52
7	16	26	35	44	54
8	17	27	36	45	55
9	19	28	37	47	56
57	66	76	85	94	104
58	68	77	86	96	105
59	69	78	87	97	106
61	70	79	89	98	107
62	71	80	90	99	108
63	72	82	91	100	110
64	73	83	92	101	111
65	75	84	93	103	112

5.3. Key Scheduling Extension

In the extended algorithm, the same style of shift and select key scheduling will be used. A new shift schedule will be designed to satisfy the known design criteria, which will involve some interaction with the design of *PC*2, as indicated before. Since sixteen rounds are planned in the extended design (provided analyses of the dependence of output bits on input and key prove satisfactory), this implies rotations greater than 2 bits in order to have a total rotation of 112 bits in sixteen rounds. This impacts on the minimum spacing requirement between bits selected in *PC*2. One such Key Schedule proposed is given in Table 11.

Table 11 - Key Schedule for an Extended DES																
Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Shift	3	3	3	4	4	4	4	3	3	3	4	4	4	4	3	3
Total	3	6	9	13	17	21	25	28	31	34	38	42	46	50	53	56

5.4. Dependency Analysis of An Extended DES

To provide an indication of the suitability of the extended rules, Meyer's dependency analysis was applied to the sample permutations developed, as well as to the worst case permutations P and PC2 (given in Tables 15 and 16). The results are given in Table 12 for Ciphertext-Plaintext dependence, Table 13 for Ciphertext-Key dependence with a worst case PC2, and in Table 14 for Ciphertext-Key dependence with the sample PC2. These results show that with the sample permutations, the dependency profile is only one round worse than in the current case (consistent with a doubling of the data lengths). With the worst case permutations however, the profile worsens significantly.

Table 12 - Dependency of Ciphertext bits on Plaintext bits in Extended (128-bit) DES		
Round	Sample XDES P	Worst XDES P
1	3.125	3.125
2	17.68	10.55
3	50.98	21.88
4	84.47	34.38
5	98.44	46.88
6	100.00	59.38
7	100.00	71.88
8	100.00	84.38
9	100.00	94.53
10	100.00	99.21
11	100.00	100.00
12	100.00	100.00

Table 13 - Dependency of Ciphertext bits on Key bits in Extended (128-bit) DES with a Worst Case PC2		
Round	Sample XDES P	Worst XDES P
1	2.68	2.68
2	19.08	11.05
3	57.37	22.94
4	89.51	35.71
5	98.44	49.33
6	99.89	63.28
7	100.00	76.12
8	100.00	87.44
9	100.00	95.76
10	100.00	99.22
11	100.00	99.89
12	100.00	100.00

Table 14 - Dependency of Ciphertext bits on Key bits in Extended (128-bit) DES with a Sample Regular PC2

Round	Sample XDES P	Worst XDES P
1	2.68	2.68
2	20.54	13.39
3	61.16	31.36
4	92.86	48.77
5	99.55	62.95
6	100.00	76.00
7	100.00	87.61
8	100.00	95.65
9	100.00	99.22
10	100.00	100.00
11	100.00	100.00
12	100.00	100.00

Table 15 - Worst Case Extended DES P

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
11	11	11	11
12	12	12	12
13	13	13	13
14	14	14	14
15	15	15	15
16	16	16	16

Table 16 - Worst Case Extended DES PC2					
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
57	58	59	60	61	62
63	64	65	66	67	68
69	70	71	72	73	74
75	76	77	78	79	80
81	82	83	84	85	86
87	88	89	90	91	92
93	94	95	96	97	98
99	100	101	102	103	104

6. Conclusion

In this paper, all the known design criteria for the S-boxes, P-boxes, and key scheduling in the Data Encryption Standard have been identified. These all appear to be related to the avalanche and completeness effects. The paper concludes with a discussion on how these criteria will be used to design an extended DES, as well as aiding in identifying further design criteria.

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